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DCJ-003-2016002 Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

July – 2022

Math-09(A) : Mathematical Analysis-II & Abstract Algebra-II

Faculty Code : 003
Subject Code : 2016002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.
(2) Figure to the right indicate full marks of the question.

1 (A) Answer the following questions in short : **4**

(1) Define : Sequentially Compact.
(2) Define : Countable set.
(3) Define : Compact set.
(4) True OR false : [0, 1] is a connected subset of $\mathbb{R}$.

(B) Answer any **one** in brief : **2**

(1) Show that (0, 1] is not a compact subset of $\mathbb{R}$.
(2) Show that $\mathbb{R}$ is not a sequentially compact.

(C) Answer any **one** in detail : **3**

(1) Show that $\mathbb{Z}$ is countable.
(2) Prove that continuous image of a compact set is compact.

(D) Answer any **one** : **5**

(1) Show that every compact subset of a metric space is bounded.
(2) Prove that every closed subset of a compact metric space is compact.

2 (A) Answer the following questions in short : **4**

(1) Find $L(t^3)$.
(2) Find $L^{-1}\left(\frac{s^2-3s+4}{s^3}\right)$.
(3) Find convolution product of $f(t)=\sin t$ and $g(t)=t$.
(4) Find $L^{-1}\left(\frac{1}{s-3}\right)$.

(B) Answer any **one** in brief : **2**

(1) Find Laplace transform of $f(t)=\begin{cases}3 & 0<t<5\\ 0 & t>5\end{cases}$.

(2) Find $L^{-1}(F(s))$. Where $F(s)=\log\frac{s+a}{s+b}$.

(C) Answer any **one** in detail : **3**

(1) Find Laplace transform of $f(t)=(\sin 2t-\cos 2t)^2$.

(2) Find inverse Laplace transform of $F(s)=\frac{3s+7}{s^2-2s-3}$.

(D) Answer any one : **5**

(1) Evaluate $\int_0^\infty e^{-2t}\sin^3 t\,dt$.

(2) Find the inverse Laplace transform of

$$\frac{s^2}{(s^2+a^2)(s^2+b^2)}.$$

3 (A) Answer the following questions in short : **4**

(1) State First Shifting Theorem for Laplace Transform.

(2) Find $L\{(t+1)^2\}$.

(3) If $L\{f(t)\}=F(s)$, then $L\left\{\frac{f(t)}{t}\right\}$ = ?

(4) State the Convolution Theorem.

(B) Answer any **one** in brief : **2**

(1) Find $L(4t^2+\sin 3t+e^{2t})$.

(2) Find $L(\sin 2t-\cos 2t)^2$.

(C) Answer any **one** in detail : **3**

(1) Prove that :

If $L\{f(t)\}=F(s)$, then $L\{f(at)\}=\frac{1}{a}F\left(\frac{s}{a}\right)$.

(2) Prove that :

If $L\{f(t)\}=F(s)$, then $L\{t^n f(t)\}=(-1)^n\frac{d^n}{ds^n}F(s)$.

(D) Answer any **one** : **5**

(1) Find $L(\cosh at \cos at)$.

(2) Using convolution theorem find $L^{-1}\left(\frac{1}{(s-2)(s+1)^2}\right)$.

4 (A) Answer the following questions in short : **4**

(1) Define : Group homomorphism.

(2) How many units in the $(\mathbb{Z}_{10}, +_{10}, \times_{10})$? List them.

(3) Let $f : G \to G; f(g) = g (g \in G)$ be a group homomorphism. Find ker f.

(4) List all the ideals of the ring $(\mathbb{Q}, +, \cdot)$.

(B) Answer any **one** in brief : **2**

(1) Does union of two subrings of a ring is a subring of the ring ? Justify.

(2) Show that in an integral domain 1 and 1 are the only idempotent elements.

(C) Answer any **one** in detail : **3**

(1) Let $f : G \to \overline{G}$ be a group homomorphism. Show that f is one-one iff ker $f = \{e\}$.

(2) Let R be a commutative ring and $a \in R$. Show that $A = \{x \in R \mid ax = 0\}$ is an ideal in R.

(D) Answer any **one** : **5**

(1) Prove that field has no proper ideal.

(2) State and prove Fundamental Theorem of group homomorphism.

5 (A) Answer the following questions in short : **4**

(1) Give an example of a non-commutative ring with unity.

(2) If $f = (2, -1, 0, 11, 0, 0, 0, ...)$ and $g = (-1, 8, 2, -2, 0, 0, 0, ...)$, then find $f + g$.

(3) True or False : Every integral domain is a field.

(4) Define : Integral domain.

(B) Answer any **one** in brief : **2**

(1) Let $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$ is a ring under usual addition and multiplication. Find inverse of $-1+2\sqrt{2}$ in $\mathbb{Q}[\sqrt{2}]$.

(2) Does $S = \{A \in M_2(\mathbb{R}) \mid \det (A) = 0\}$ is subring of $(M_2(\mathbb{R}), +, \cdot)$? Justify.

(C) Answer any **one** in detail : **3**

(1) Show that the polynomial $f(x) = 8x^3 + 6x^2 - 9x + 24$ is irreducible over $\mathbb{Q}$.

(2) Prove that for any prime p, the p^{th} cyclotomic polynomial

$$\Phi_p(x) = \frac{x^p - 1}{x - 1} = x^{p-1} + x^{p-2} + \ldots + x + 1$$

is irreducibel over $\mathbb{Q}$.

(D) Answer any **one** : **5**

(1) If F is a field, then show that $F[x]$ is never a field.

(2) State and prove division algorithm in $F[x]$, where F is a field.